LA-8410-MS

Informal Report

C.3

Thickening of a Shock Front by the Grain Structure of a Solid

University of California

LOS ALAMOS SCIENTIFIC LABORATORY
Post Office Box 1663 Los Alamos, New Mexico 87545

This report was not edited by the Technical Information staff.

UNITED STATES
DEPARTMENT OF ENERGY
CONTRACT W-7405-ENG. 36

LA-8410-MS
Informal Report

UC-34
Issued: July 1980

Thickening of a Shock Front by the Grain Structure of a Solid

R. C. Mjolsness

THICKENING OF A SHOCK FRONT BY THE GRAIN STRUCTURE OF A SOLID

by

R. C. Mjolsness

ABSTRACT

Three possible mechanisms, (1) microchanneling of a shock front, (2) variations of sound speed with direction in a grain, (3) refraction of shock front direction on passing from grain to grain, all seem able to cause an initially steady, 1-D shock front to effectively thicken into a fluctuating, irregular shock front described by a shock front envelope. The shock front envelope diffuses in width to the order of one to several grain diameters, when it presumably reaches a steady state width. The treatment is confined to estimating the initial rate of widening of the shock front envelope by the simplest reasonable models, so no properties of the conjectured final state shock structure have been established.

I. INTRODUCTION

The propagation of a shock wave through a homogeneous medium may be idealized as a one-dimensional, steady state process (in the shock frame) in which the thickness of the shock front is determined by dissipative processes. The shock front thickness is usually very small. For example, in a gas heat conduction and viscous forces cause the thickness to be of the general order of the mean free path of the gas.

The question was raised by D. J. Sandstrom[1] whether, when a shock propagates through a solid which, due to its grain structure, is intrinsically heterogeneous in its spatial microstructure, there might be physical processes which would tend to thicken and/or roughen a shock front beyond the values to be expected for propagation in a truly homogeneous medium. A definitive theoretical treatment of this topic, taking full account of the microstructure of a solid, is extremely difficult and is not attempted here. Instead, we consider three possible mechanisms which could lead to the shock front actually being an envelope of spatially and temporally irregular local shock fronts, the half-width of the envelope increasing with propagation distance through the solid. Presumably, the envelope half-width stabilizes at a thickness of the order of one to several times the average grain size, since the medium looks homogeneous on scales appreciably larger than this. However, no trace of such a saturation effect can be found in our models, which are confined to treating by the very simplest reasonable means the initial phase of the thickening of the envelope of the local shock front, showing that there is a tendency to thicken and estimating the initial rate of such thickening. We make heavy, but apparently reasonable, use of assumptions of the statistical independence of numerous, small impulses to treat the effects of repeated, local interactions of the shock front with the heterogeneous medium as a Markovian process of random walk character. We believe the treatment suffices to show that there are physical mechanisms which tend to thicken the envelope of shock front locations.

We consider three possible mechanisms for thickening the shock front, i.e., for thickening the spatial envelope that contains the irregular locations of the local shock front;

(1) Microchanneling of shock front in interstices between grains

(2) Variations of sound speed with direction inside a grain

(3) Refraction of shock front direction on passing through a grain.

Microchanneling would be a very effective process for thickening the shock front envelope in materials for which a significant fraction, such as 1%, of the mass exists outside of distinct grains or in materials with a wide distribution of grain sizes, in which the large grains tend to be separated by numerous small grains. However, we understand that normally grains tend to be contiguous down to lattice dimensions. In addition, we are uncertain whether there is commonly a significantly different environment (of small grains) between neighboring large grains. Thus we are dubious of the physical importance of this mechanism, though we are able to estimate its effect (large) on the width of the shock front envelope, <u>on the assumption that the mechanism exists</u>.

Variations of sound speed with direction of propagation in a grain, if it exists, would also provide a thickening of the shock front envelope as the shock front propagates through a solid. We do not know whether such a variation of sound has been observed, but we see no particular reason to doubt that it might exist. However, for seemingly reasonable postulated variations of sound speed, it turns out that the shock must traverse tens of thousands of average grain diameters before the shock front envelope would be as thick as an average grain diameter. Thus, for propagation paths of centimeters or tens of centimeters through a solid, this mechanism would not be very effective, and we don't believe that this mechanism is likely to be the dominant one.

Finally, the refraction of the local angle of propagation of the shock front upon passing through a grain is a physical effect that almost certainly exists and the correlation of net angles of propagation after N "scattering" events results in an extremely rapid build-up of shock front envelope thickness. Our calculational technique seriously overestimates this thickening for events in which the local front propagates at a large angle ($\gtrsim$ one radian) to the average angle of propagation for an appreciable fraction of the time, but it should suffice to indicate reasonable initial rates of thickening. In addition, we do not expect significant error to arise from this source because the shock front envelope reaches its final thickness long before the angular errors are appreciable, as discussed in Section. V. This mechanism seems efficient and it almost certainly exists. Thus it may well be the dominant mechanism, particularly if microchanneling can not occur.

We note that HE driven shocks propagating through a solid commonly have shock pressures of several hundred kilobars and commonly eject "fluff" when the shock emerges from the surface into gas or into vacuo. It is not clear that the mechanisms responsible for fluff ejection have been completely identified, but some of the fluff may be due to a spalling of particles from the surface. It seems likely that an irregular shock front of the type discussed here, whose shock front envelope is of the order of a grain size, would have a significant effect on the amount of material ejected by spall, since it could put pressure differences of tens of kilobars across an individual grain near or at the surface of the metal.

II. STATISTICAL DESCRIPTION OF SHOCK FRONT LOCATION

We consider each element of surface area of the shock front (of roughly an average grain size) to propagate independently, subject to a small random influence (produced by one of the three mechanisms being considered; we do not treat the joint action of several of the mechanisms) every time the shock front traverses an average grain size in the solid. We neglect the intrinsic width of the shock front and, for maximal simplicity, consider grains to be all of the average size a, but to have random orientations. The random influence affects directly either the distance the surface element has propagated (in microchanneling and in sound speed variations) or the angle with respect to the average direction of propagation at which the surface element is propagated locally (in refraction). Of course, the variations in angle of propagation also give rise to variations in the distance the surface front has propagated in the direction of average propagation (which we take here to be the x axis). On the average, these variations in propagation distance in the x direction must be zero. That is, any surface element is, on the average, at the average position of the shock front, which is assumed to be simply the position of a 1-D, steady shock front propagating in the x direction.

Thus the picture we form of the shock propagation process is that of a wrinkled irregular (over dimensions of a grain size) shock front, whose surface elements are subject to repeated random influences on passing through grains but which are, on the average, to be found at the position of an idealized 1-D, steady shock front. Each surface element has a time dependent probability distribution of being a distance $\Delta x(t)$ from the average position of the shock front, and we characterize this probability distribution very crudely by calculating the shock front envelope, which we take to be a pair of planes parallel to the plane of the average, 1-D shock front and separated from this shock front by the RMS distance, $\sigma(t) = \langle \Delta x^2(t) \rangle^{1/2}$.

We presume $\sigma(t)$ is the same for any surface element and calculate it by presuming that Δx is built up in N discrete steps, separated by time interval Δt, after each shock traversal of a grain size a. That is, we calculate $\sigma_N \equiv \sigma(t_N) \equiv \sigma(N\Delta t)$ from the usual statistical assumptions for Markovian processes involving

$$\Delta x_N = \sum_{i=1}^{N} \delta x_i \quad , \tag{1}$$

in which δx_i is the stochastic variable displacement (relative to the average shock front position) suffered by the surface. The statistical properties of the shock refraction mechanism are somewhat different because the random deviations in propagation direction $\delta\Theta_i$ are the stochastic variables and δx_i is non-linear (to first approximatation, quadratic) in $\delta\Theta_i$.

With this framework, the evaluation of σ_N is possible, and is even simple. This should not be allowed to obscure the fact that the underlying physical situation that is being approximated is incredibly complex. Thus the minimal ingredients for an adequate theoretical treatment of this topic would seem to be a local and instantaneous description of each surface element of the shock front, an instantaneous (presently unavailable) evaluation of the local shock front position on passing through a grain of given size, shape, orientation and physical properties, a (mostly unavailable) description of the statistical distribution of grain properties and an (unavailable) tabulation of relevant physical properties, such as a possible dependence of sound speed on orientation of the grain, of the various grains. We are far from attaining such an adequate theory.

The present results are presented here primarily because they apparently indicate clearly that any such random influence on the shock front will produce a half-width σ_N for the shock front envelope that increases as the shock propagates through the solid and because it seems physically reasonable that there will be such 'random' influences in propagation through a heterogeneous medium. Thus one might have some willingness to attach possible signifiance to the physical picture of the shock front envelope, that emerges so clearly from the present calculations, even in the absence of an adequate theory supporting and developing the picture.

III. MICROCHANNELING

We assume that a given surface element has a probability $g < 1$ for being in a channel between grains, or in a region of small grains between two large grains in the i^{th} grain traversal of the shock front. The effect of being in a channel is assumed to be analogous to the macroscopic propagation of a shock through a channel of variable dimensions and, possibly, composition. Specifically we assume that in the channel there are accelerations and decelerations of the shock propagation speed such that during the time the shock traverses the i^{th} grain dimension it suffers a displacement $\delta x_i = f_i a$ with respect to the average position of the shock front. The dimensionless variable f_i has a probability distribution $P(f_i)$ which we characterize by the dimensionless RMS displacement

$$F^2 = \int df_i f_i^2 P(f_i) \equiv \overline{f^2} \quad . \tag{2}$$

Of course, we must have

$$0 = \int df_i f_i P(f_i) \equiv \overline{f} \quad , \tag{3}$$

simply from the definition of the average position of the shock front. When the surface element is not in the channel, the displacement of the surface element with respect to the average position of the shock front is taken to be zero; i.e.,

$$\delta x_i = 0 \quad . \tag{4}$$

As stated earlier, we are far from convinced that this mechanism is an important physical process. What we do here is give a very simple evaluation of the process which appears to indicate that this mechanism, if it exists, will be a very efficient disperser of the shock front if one assumes apparently reasonable values for g and F.

To formalize the calculation we introduce a second random variable $q_i (0 \leqslant q_i \leqslant 1)$ for the i^{th} traversal of the shock front through a grain size in order to describe the probability that the surface element is in a channel during the i^{th} traversal. In the present approximation, the statistical properties of each surface element are identical. Thus, we will consider only one such surface element and use it to evaluate

$$\sigma_N = < \Delta x_N^2 >^{\frac{1}{2}} \quad , \tag{5}$$

where

$$\Delta x_N = \sum_{i=1}^{N} \delta x_i(q_i, f_i) \quad , \tag{6}$$

$$\delta x_i = f_i \, a \qquad \text{for } 0 \leqslant q_i \leqslant q$$

$$= 0 \qquad \text{for } q < q_i \leqslant 1 \quad , \tag{7}$$

and

$$\langle \ldots \rangle = \prod_{i=1}^{N} \left(\int_o^1 dq_i \int df_i P(f_i) \right) \ldots \quad , \tag{8}$$

and integrals over the $j^{\underline{th}}$ probability distribution do not affect δx_i when $i \neq j$. Thus,

$$\sigma_N^2 = \prod_{i=1}^{N} \int_o^1 dq_i \int df_i P(f_i) \sum_{j,k=1}^{N} \delta x_j(q_j, f_j) \delta x_k(q_k, f_k)$$

$$= \sum_{i=1}^{N} \int_o^1 dq_i \int df_i P(f_i) \delta x_i^2(q_i, f_i)$$

$$= q \, a^2 \sum_{i=1}^{N} \int df_i P(f_i) f_i^2 \quad , \tag{9}$$

and we have

$$\sigma_n = (qF^2N)^{\frac{1}{2}} a \quad . \tag{10}$$

We then find that the thickness of the shock front envelope is of the order of the grain size,

$$\sigma_N \gtrsim \tfrac{1}{2} a \quad , \tag{11}$$

when

$$q F^2 N \gtrsim \tfrac{1}{4} \quad . \tag{12}$$

If, for example, it is reasonable to take $q \sim 10^{-2}$, $F \sim 0.2$ then we would find $\sigma_N \sim \frac{1}{2} a$ after the shock front traverses roughly 1000 grain diameters. This is a very efficient dispersion (and thickening) of the shock front and, for media with $a \sim 10^{-2}$ cm, would occur after roughly 10 cm of shock propagation. Of course, there may turn out to be no effective channeling mechanism ($g \simeq 0$), in which case no shock front envelope thickness is predicted from our results.

IV. VARIATION OF SOUND SPEED WITH DIRECTION

If the sound speed in an individual grain should vary with the direction of sound propagation relative to the orientation of the grain, then this, too, would be a mechanism for thickening the shock front envelope. To investigate this mechanism crudely, we assume that the grains are axially symmetric, with a principal axis which lies at a random angle θ to the direction of propagation of the shock front, and that the speed of sound propagating in the direction θ is

$$c(\theta) = c_{min} + \Delta c \cos^2\theta \quad , \tag{13}$$

with

$$c_{max} = c_{min} + \Delta c \quad . \tag{14}$$

For a shock being driven by a pressure p, this implies that the shock speed in the direction θ is

$$v(\theta) = v_{min} + \Delta v \cos^2\theta \tag{15}$$

with

$$v_{max} = v_{min} + \Delta v \quad . \tag{16}$$

The angle θ may be taken to be a random variable varying between 0 and $\pi/2$ with probability

$$P(\theta)d\theta = \frac{2}{\pi} d\theta \quad . \tag{17}$$

Thus the average speed of propagation of the shock front is

$$\bar{v} = v_{min} + \frac{1}{2} \Delta v \quad . \tag{18}$$

We treat the retardations and accelerations of a single surface element of the shock front by assuming that after each time interval $\Delta t = a/\bar{v}$ the surface element has traversed another grain and has suffered another displacement relative to the average position of the shock front. Specifically, after the i^{th} time interval we take

$$\delta x_i(\theta_i) = (v-\bar{v})\Delta t = \left(\frac{\Delta v}{\bar{v}}\right) a \left(\cos^2\theta_i - \frac{1}{2}\right) \quad . \tag{19}$$

The probability weighting of the set of displacements is

$$\langle \ldots \rangle = \prod_{i=1}^{N} \int P(\theta_i) d\theta_i \cdots \quad , \tag{20}$$

so we evaluate the half-thickness of the shock envelope, σ_N, from

$$\begin{aligned} \sigma_N^2 &= \prod_{i=1}^{N} \int P(\theta_i) d\theta_i \sum_{j,k=1}^{N} \delta x_j(\theta_j) \delta x_k(\theta_k) \\ &= \sum_{i=1}^{N} \int d\theta_i P(\theta_i) \delta x_i^2(\theta_i) \\ &= \frac{2}{\pi} \left(\frac{\Delta v}{\bar{v}}\right)^2 a^2 N \int_0^{\pi/2} d\theta \left(\cos^4\theta - \cos^2\theta + \frac{1}{4}\right) \quad . \end{aligned} \tag{21}$$

Thus we obtain

$$\sigma_N = \frac{1}{2\sqrt{2}} \left(\frac{\Delta v}{\bar{v}}\right) N^{\frac{1}{2}} a \quad . \tag{22}$$

The shock front envelope thickens as $N^{\frac{1}{2}}$ in this case also. But if it is reasonable to estimate a 1% variation in sound speed, i.e., that $\Delta v/\bar{v} \sim \frac{1}{2} \times 10^{-2}$, then it would require $N \sim 80,000$ grain traversals of the shock front to obtain Eq. (11), specifying that the shock front envelope thickness is of the order of the grain size. Very often, there will not be that many grain widths in the sample of material being shocked.

V. SHOCK REFRACTION

A surface element probably refracts somewhat in propagation direction on passing from grain to grain. Any intrinsic variability of medium properties from grain to grain would contribute to such an effect. In addition, any variation of medium properties, such as sound speed, with orientation of direction of propagation relative to the major axis of the grain would yield such an effect. For example, we may obtain order of magnitude estimates of the expected size of effects if we idealize the shock passage from grain to grain as the oblique passage of a shock through two contiguous slabs of polytropic gases of different densities and polytropic indices, with the Grüneisen coefficient $\Gamma = \gamma - 1$. Then a 1% variation in medium properties would yield on the average a refraction of propagation angle $\delta\theta \sim 10^{-3}$ radian.

The evaluation of the dispersion of the shock front envelope is very difficult in general due to the partial correlation in propagation direction upon successive collisions, the detailed nature of the statistical sums and the mathematical representation of the resulting σ_N being very strongly dependent on the RMS scattering angle σ, where

$$\alpha^2 = < \delta\theta^2 > \quad . \tag{23}$$

Only two cases seem at all tractable to a simple analysis: a large refraction angle model applicable when $\alpha \gtrsim 1$ (radian), and possibly of some validity when $\alpha \gtrsim 10^{-1}$, and a small refraction angle model which may be valid for $\alpha \lesssim 10^{-2}$ (but possibly only for much smaller angles). We evaluate σ_N for the two models below.

For both models we evaluate σ_N for a single surface element on the assumption that it passes successively through N grains which have random orientation with respect to the original direction of propagation (x axis). In the $i^{\underline{th}}$ grain the direction of propagation makes an angle θ_i with respect to the x axis. The shock initially propagates with velocity v, but due to the random refractions in angle of the various surface elements its average velocity of propagation in the x direction $\bar{v}(i)$ is a weak function of i, and decreases slightly as i increases. Each passage of the grain shock through a grain takes roughly a time $\Delta t = a/v$, during which the shock surface element is displaced

$$\delta x_i = \Delta t(v \cos\theta_i - \bar{v}_i) \tag{24}$$

relative to the average position of the shock front. We describe this displacement via the approximation

$$\delta x_i = a\left[\left(\frac{v-\bar{v}_i}{v}\right) - \frac{1}{2}\,\mathit{0}_i^2\right] \quad , \tag{25}$$

which implies that our estimates for σ_N will greatly overestimate the spread of the shock front envelope at very large N, or whenever the average value of $\mathit{0}_i^2$ becomes very large. However, this does not occur until long after the shock front envelope reaches its (conjectured) final thickness $\sigma_N \gtrsim a$. Our theory gives the initial rate of spreading of σ_N because it contains no mechanism for slowing the growth of σ_N as $\sigma_N \sim a$. We assume that the surface element suffers a refraction of propagation angle $\delta\mathit{0}_i$ in passing from the (i-1)st to the ith grain. Thus, in general,

$$\mathit{0}_i = \mathit{0}_{i-1} + \delta\mathit{0}_i = \sum_{j=1}^{i} \delta\mathit{0}_j \quad . \tag{26}$$

Each random variable $\delta\mathit{0}_i$ is taken to be an independent normally distributed variable with variance α, i.e.,

$$< \delta\mathit{0}_i^2 > = \alpha^2 \quad , \tag{27}$$

and

$$< \delta\mathit{0}_i^4 > = 3\alpha^4 \quad , \tag{28}$$

where

$$<\ldots> = \prod_{j=1}^{N} \int d\mathit{0}_j \ \cdots \quad , \tag{29}$$

for a passage through N grains. The models differ in how the statistical properties of $\mathcal{O}_i$ are to be treated.

Model A: Large Refraction Angles

For this case it is reasonable to treat $\mathcal{O}_i$ as dominated by the last random impulse $\delta\mathcal{O}_i$ and take $\mathcal{O}_i$ to be itself the normal random variable of variance α. Since we must have

$$< \delta x_i > = 0 \quad , \tag{30}$$

by definition, $\bar{v}_i$ is given by

$$\bar{v}_i = v[1 - \frac{1}{2}\alpha^2] \quad . \tag{31}$$

The average velocity is decreased by collisions but in this case the absence of correlations between successive $\mathcal{O}_i$ prevents $\bar{v}_i$ from decreasing as i increases. The shock front envelope half width σ_N is given by

$$\sigma_N^2 = \frac{1}{4} a^2 \left\langle \left(\sum_{i=1}^{N} \left(\alpha^2 - \mathcal{O}_i^2 \right) \right)^2 \right\rangle$$

$$= \frac{1}{4} a^2 \left(\left\langle \sum_{i=1}^{N} \sum_{j=1}^{N} \delta\mathcal{O}_i^2 \delta\mathcal{O}_j^2 \right\rangle - N^2\alpha^4 \right) \quad . \tag{32}$$

The statistical sum is evaluated through

$$\left\langle \sum_{i=1}^{N} \sum_{j=1}^{N} \delta\mathcal{O}_i^2 \delta\mathcal{O}_j^2 \right\rangle = \sum_{i=1}^{N} \delta\mathcal{O}_i^4 + \left\langle \sum_{\substack{i,j=1 \\ i\neq j}}^{N} \delta\mathcal{O}_i^2 \delta\mathcal{O}_j^2 \right\rangle$$

$$= 3N\,\alpha^4 + \left\langle \sum_{i=1}^{N} \delta\theta_i^2 \right\rangle \left\langle \sum_{\substack{j=1\\ j\neq i}}^{N} \delta\theta_j^2 \right\rangle$$

$$= 2N\,\alpha^4 + \left\langle \sum_{i=1}^{N} \delta\theta_i^2 \right\rangle \left\langle \sum_{j=1}^{N} \delta\theta_j^2 \right\rangle$$

$$= 2N\,\alpha^4 + N^2\alpha^4 \quad . \tag{33}$$

Thus

$$\sigma_N \approx \frac{1}{\sqrt{2}}\, N^{\frac{1}{2}}\alpha^2\, a \quad . \tag{34}$$

For this case we probably need $\alpha < 1$ for the small angle approximation of Eq. (25) to be acceptable, but, of course, the smaller that α is the less credible is the treatment of the refraction processes specified by this model. Because we think α is likely to be small we doubt that this model is directly applicable to shock refraction in a solid. Rather its interest is as a suggestive limiting case, which, because of the large values of α needed in this regime, suggests an efficient diffusion of the shock front envelope. For example, if $\alpha^2 = 10^{-1}$ then after the shock traverses only $N \sim 100$ grain diameters the shock front envelope is predicted to be the width of the grain size a. Even at the limit of applicability of the model $\alpha = 10^{-1}$ this diffusion would be complete by $N = 10^4$ grain traversals.

Model B: Small Refraction Angles

For this case we accept the strong correlations from past refraction process implied by Eq. (26) and treat each $\delta\theta_i$ as an independent random process, as specified by Eqs. (27)-(29). We show in Appendix A that this leads to the extremely rapid build-up of σ_N with N specified for large N by

$$\sigma_N \simeq \frac{1}{2\sqrt{3}}\, N^2\alpha^2 a \quad . \tag{35}$$

The correlation among propagation angles also implies that the average shock propagation velocity decreases weakly, but progressively, with i, according to

$$\bar{v}_i = \left[1 - \frac{1}{2} i \alpha^2\right] v \quad , \tag{36}$$

which is another testable prediction of the model.

The very different N dependence of Eq. (35) comes from the quartic sums over refraction angles implicit in evaluating σ_N through the highly correlated expressions of Eqs. (25) and (26). The dispersion of individual random variables makes a contribution of order $N^{3/2}$ to δ_N, rather than of order $N^{1/2}$ as in the several previous models. The calculation is, however, dominated in this case by interference terms in which there are two distinct pairs of scattering angles. These terms interfere constructively and yield the contribution of order N^2.

For this case α is small, but the strong N dependence of σ_N still yields an efficient diffusion of the shock front envelope. For example, if $\alpha = 10^{-2}$ the shock front envelope will be of the grain size a after a shock propagation of only $N \sim 200$ grain diameters. Even if $\alpha = 10^{-3}$ this dispersion will occur after only $N \sim 2000$ grain diameters.

We should point out that the dispersion limit $\sigma_N \sim a$ occurs long before the small angle requirement of the theory, measured, for example, by

$$< \theta_N^2 > = N\alpha^2 < 1 \quad , \tag{37}$$

becomes invalid. This is because the shock front envelope is of the order of the grain size a when

$$N^2\alpha^2 \sim 1 \quad . \tag{38}$$

For small α, Eq. (37) is well satisfied when Eq. (38) holds.

We presume that the correct treatment of correlations between refraction events is largely set by the size of α. It seems reasonable to presume that in general

$$\sigma_N(\alpha) \sim N^{n(\alpha)}\alpha^2 a \quad , \tag{39}$$

with

$$n(1) = \frac{1}{2}$$

and

$$n(0) = 2 \quad . \tag{40}$$

From the N values predicted from the limiting models A and B for diffusion of the shock front envelope to a grain size thickness, it seems likely that in general this process will be efficient. Numerical values in these models were obtained through the implicit assumption that in Eq. (37)

$$n(0.1) \sim \frac{1}{2}$$

and

$$n(10^{-2}) \sim 2 \quad . \tag{41}$$

The major variation of $n(\alpha)$ was thus presumed to occur between $\alpha = 10^{-2}$ and $\alpha = 10^{-1}$.

VI. POSSIBLE LIMITING WIDTH OF SHOCK FRONT ENVELOPE

We have stated that it is probable that the shock front envelope achieves a limiting half-width σ_∞ of the order of one to several grain lengths a, because the medium is roughly homogeneous, on length scales larger than this. Our treatment of the heterogeneous dynamics, applicable only to the initial spreading phase of the shock front envelope, indicates a wrinkled, irregular, time dependent shock front bounded by the regular shock front envelope. Such a picture would suggest an irregular, time dependent shock front at all times, with only the limiting shock front envelope half-width σ_∞ becoming static. But we can not be sure that this is a correct inference from the model. It is also conceivable that the entire shock front approaches a steady, 1-D structure of the order of σ_∞ in width, the structure being stable to the very small perturbations being continuously excited by the heterogeneity of the medium, the perturbations becoming increasingly small as $\sigma(t) \to \sigma_\infty$. Thus, only in the initial, spreading phase of the shock front envelope can we be sure that the shock front is an irregular, time dependent structure.

The existence of a steady, 1-D limiting shock front width would have a dramatic effect on the amount of spall produced when the shock emerges from the metal surface. There would then be a "worst" distance of propagation through the metal, before emergence from the surface, at which the spall produced would be maximum. Study of the variation of spall production versus distance of shock propagation might be an experimental technique for testing whether such a steady, 1-D limiting shock front structure exists.

VII. CONCLUSIONS

While the theory given here is extremely crude, we believe that it does make plausible the notion that definite physical mechanisms will cause a shock front propagating through a heterogeneous solid to not be a steady state, 1-D structure but will instead force an irregular, variable structure whose shock front envelope thickens, at least initially, as the shock propagates through the solid. It seems reasonable that such irregular structures would have a significant influence on the amount of fluff ejected by spall mechanisms. We doubt that the irregular structure would greatly influence the amount of fluff ejected via microjetting, since even the irregular structures would be reasonably smooth and flat over the dimensions (typically ~ microns) involved in surface irregularities. However, calculations indicate that microjetted material is ejected at two to three times the speed of the surface while VISAR data apparently indicate that the bulk of the fluff is ejected at substantially lower speeds. This suggests that spall processes may dominate fluff ejection. Thus it is important to investigate processes, like the present ones, that enhance spall.

In particular, it would seem justifiable to undertake careful experimental measurements of the thickness, and of the regularity of a shock front propagating through a heterogeneous solid. The importance of tackling this question experimentally is further enhanced by the fact that a reliable and definitive theoretical treatment of such problems in heterogeneous media would seem to be extremely difficult. Impressive theoretical treatments of simplified physics in shock propagation through heterogeneous materials already exist,[2] but treatments which fully account for the complexity of the underlying heterogeneous phenomena may not be available for some time.

ACKNOWLEDGMENTS

The author would like to thank Dr. D. J. Sandstrom for the initial inquiry that led to this work and Drs. J. K. Dienes and H. M. Ruppel for very helpful discussions.

REFERENCES

1. D. J. Sandstrom, (private communication).

2. T. L. Cook, R. B. Demuth and F. H. Harlow, "Multiphase Interpenetration of Shocked Materials," Los Alamos Scientific Laboratory report LA-7578 (1979).

APPENDIX

EVALUATION OF STATISTICAL SUMS OF MODEL B

Here we evaluate σ_N and $\bar{v}_i$ for model B. From Eqs. (25) and (26) we obtain

$$\delta x_i = \left(\frac{v-\bar{v}_i}{v}\right) - \frac{1}{2} \sum_{j=1}^{i} \sum_{k=1}^{i} \delta\theta_j \delta\theta_k \quad . \tag{A-1}$$

From Eq. (30) this yields

$$\frac{v-\bar{v}_i}{v} = \frac{1}{2}\, i\, \alpha^2 \tag{A-2}$$

or

$$\bar{v}_i = \left[1 - \frac{1}{2}\, i\, \alpha^2\right] v \quad . \tag{A-3}$$

From the defining Eq. (1) for Δx_N we obtain

$$\Delta x_N = \frac{1}{4}\left[N(N+1)\alpha^2 - 2A\right] a \quad , \tag{A-4}$$

where

$$A = \sum_{j=1}^{N} \sum_{k=1}^{N} c_{jk} \delta\theta_j \delta\theta_k \tag{A-5}$$

and

$$c_{jk} = N+1 - \max(j,k) \quad . \tag{A-6}$$

We then evaluate σ_N from

$$\sigma_N^2 = \frac{1}{16}\left[4 < A^2 > - N^2(N+1)^2\alpha^4 \ a^2\right] \quad . \tag{A-7}$$

We must deal with quartic statistical sums in evaluating

$$\begin{aligned} < A^2 > &= \sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{m=1}^{N}\sum_{n=1}^{N} c_{jk}c_{mn} < \delta\theta_j\delta\theta_k\delta\theta_m\delta\theta_n > \\ &= \sum_{j=1}^{N} c_{jj}^2 < \delta\theta_j^4 > + \sum_{\substack{j,m=1 \\ j\neq m}}^{N}\sum c_{jj}c_{mm}\,\alpha^4 \\ &+ 4 \sum_{\substack{j,k=1 \\ j>k}}^{N} c_{jk}^2\,\alpha^4 \\ &= \alpha^4 \Bigg\{2 \sum_{j=1}^{N} [(N+1)^2 - 2(N+1)j + j^2] \\ &+ \sum_{j=1}^{N}\sum_{m=1}^{N} [(N+1)^2 - (N+1)(j+m) + jm] \\ &+ 4 \sum_{j=1}^{N} [(N+1)^2 - 2(N+1)j + j^2](j-1)\Bigg\} \quad . \end{aligned} \tag{A-8}$$

Fortunately, the sums are all elementary and yield for the three terms

$$< A^2 > = \alpha^4\left\{\frac{1}{3} N(N+1)(2N+1) + \frac{1}{4} N^2(N+1)^2 + \frac{1}{3} N^2(N^2-1)\right\} \quad . \tag{A-9}$$

The first term arises from the dispersion of the random variables $\delta\theta_j$, the second is the dispersionless contribution cancelled by $< A >^2$ in Eq. (A-7), while the third term arises from constructive interference terms in which two pairs of angles are equal. We then obtain

$$\sigma_N^2 = \frac{1}{12} N(N+1)(N^2 + N + 1)\alpha^4 a^2 \quad , \tag{A-10}$$

which yields the results of the text.

Printed in the United States of America. Available from
National Technical Information Service
US Department of Commerce
5285 Port Royal Road
Springfield, VA 22161

Microfiche $3.00

001-025	4.00	126-150	7.25	251-275	10.75	376-400	13.00	501-525	15.25
026-050	4.50	151-175	8.00	276-300	11.00	401-425	13.25	526-550	15.50
051-075	5.25	176-200	9.00	301-325	11.75	426-450	14.00	551-575	16.25
076-100	6.00	201-225	9.25	326-350	12.00	451-475	14.50	576-600	16.50
101-125	6.50	226-250	9.50	351-375	12.50	476-500	15.00	601-up	

Note: Add $2.50 for each additional 100-page increment from 601 pages up.